Poissonian Statistics in Gamma Radiation

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We examine the Poisson statistics of gamma radiation using a Cesium-137 source and a photomultiplier setup. The experimental setup was arranged to provide approximate radiation count rates of ∼1Hz, ∼5Hz, ∼10Hz, and ∼100Hz; then, for each arrangement, we count gamma ray hits in 1s intervals and 100s intervals. The resulting data appears strongly Poissonian for the higher rates. We perform a full statistical analysis, and then suggest sources of error to account for the discrepancies at the 1Hz level.

Poisson statistics are ubiquitous in the physical sciences, so we will first lay the groundwork with a mathematical discussion of the distribution, before applying it to gamma radiation.

I. POISSON STATISTICS

The origin of the Poisson Distribution is the binomial distribution, which arises from the following question: *Suppose we have a series of n independent trials, each having the same probability p of success. What is the probability of finding k successes?*

For instance, suppose we flip n coins ($p = .5$), and count the number k of heads. If we repeat the entire procedure many times, what is the distribution of k values? The answer is simply the probabilty of succeeding in k trials times the probability of failing in $n-k$ trails times the number of ways of reordering those indistinguishable successes and failures:

$$P(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

Intuitively, the mean number of successes is the probability times the number of trials: $\mu_k = np$, and the distribution is peaked around this value.

Now suppose, instead of having n discrete trials with fixed probability p, we have a continuous procedure with a fixed mean rate of "successes." An example of this is radiation counts from a decay: it is random, but has a fixed mean rate (assuming the observation time is small compared to the half-life). We can view this is as a limit as the number of trials, n, grows to infinity (but keeping the expected count $\lambda = np$ finite). Taking this limit on the binomial distribution yields the Poisson distribution:

$$P(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

We will note many useful properties of this curve.

- There is only one parameter, λ, to the distribution.

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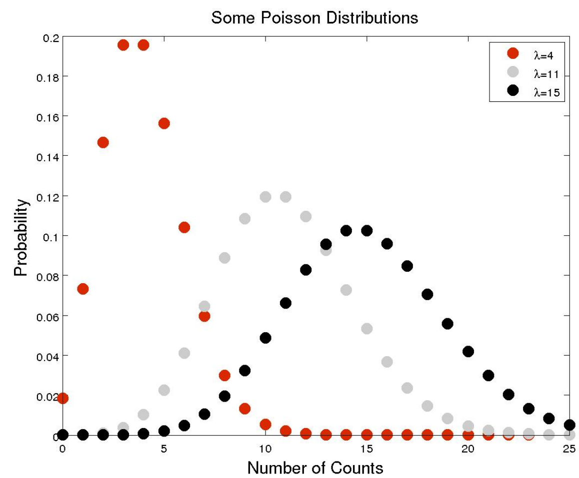

FIG. 1: Shown are three Poisson distributions. Notice that each is peaked near its λ, with a width proportional to $\sqrt{\lambda}$. Furthermore, if one measures a process which is the sum of the processes producing the first two curves ($\lambda = 4, 11$), the result will be described by the third curve ($\lambda = 15$).

- The mean, variance (and, in fact, all higher moments) are λ.
- If two Poissonian processes (with mean rates λ_1 and λ_2) contribute counts to an experiment, then the resulting distribution is also Poissonian, with mean rate $\lambda = \lambda_1 + \lambda_2$.

Examine Figure 1 for examples of Poisson curves. To see this distribution in nature, we arranged an experiment to count gamma rays from a radioactive source of Cesium-137.

II. EXPERIMENTAL SETUP

The radiation from the ^{137}Cs source is detected by an 802 Canberra Scintillation Detector, as depicted in Figure 2. The detector contains a small crystal of NaI, which absorbs the gamma radiation and reemits the energy as visible photons. These photons then interact with a photocathode in the photomultiplier tube (PMT), releasing

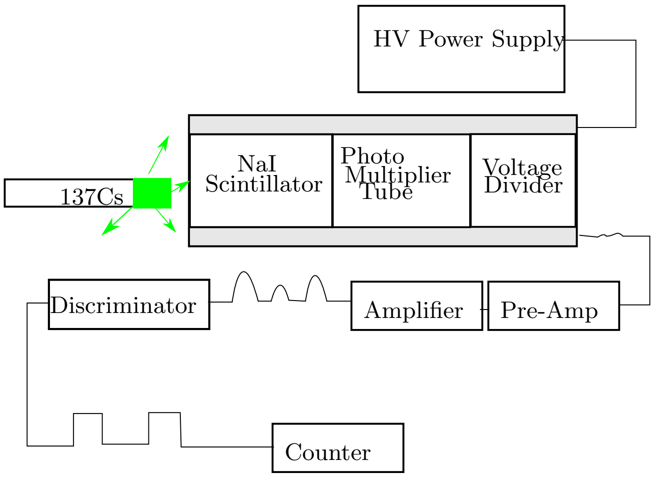

FIG. 2: Experimental setup for gamma ray detection

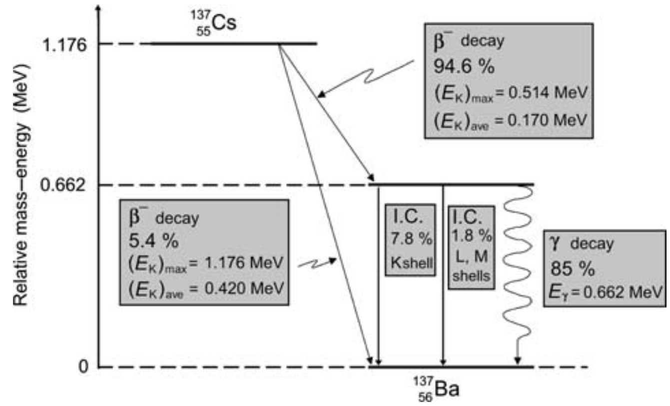

FIG. 3: Decay path of Cesium-137. Note the single γ emission at 662keV.

FIG. 4: The radiation spectrum with and without the source, respectively. Notice that the strong feature disappears when the source is removed.

a stream of electrons. A ($\sim$ 1kV) potential difference forces these photoelectrons down the body of the photomultiplier tube, which consists of a sequence of metal surfaces at increasing potential. An electron picks up kinetic energy as it falls through the potential, so that each collision with a metal sheet produces more and more freed electrons, which continue multiplying in this manner until an appreciable current reaches the voltage divider at the end of the tube.

The signal is then amplified (to the scale of about 3V) and passed to a discriminator. The discriminator converts any pulse above a given trigger level to a square bump. Since the height of the original pulse increases with the energy of the gamma ray photon which set it off, the selective discriminator passes only events from gamma rays above some adjustable energy level. The train of square pulses is then fed to the counter, from which the experimenter can read off the number of events in a given interval.

For diagnostic purposes, an oscilloscope monitors the output of both the amplifier and the discriminator. The oscilloscope triggers off the discriminator, so that we can view all of the pulseforms which get counted. When the source is in place in front of the detector, we see a bold feature on the oscilloscope, along with a lighter spectrum of pulses due to background (see Figure 4). The bold feature disappears when the source is removed, which suggests that it corresponds to the single γ emission in the ^{137}Cs decay path (see Figure 3).

III. DATA

We adjusted the discriminator to acheive various approximate count rates for the experiment: $\sim$1Hz, $\sim$5Hz, $\sim$10Hz, and $\sim$100Hz. At each desired approximate rate, we counted gamma ray hits in one hundred 1-second intervals. By running a *cumulative average* along the one hundred data points, we can graphically examine how our estimate of the mean converges upon its final value. The red plots in Figure 5 are cumulative averages, that is, the height of the plot at the n-th point represents the average, μ_n, taking into account only the first n trials. The uncertainties are calculated for each point as $\sqrt{\mu_n/n}$. On the plot, we see the means converging to final estimates with increasing precision, which is precisely what we should expect for a truly random process.

Additionally, at each desired rate, we ran a single 100-second interval as another estimate of the mean. If the count in 100s is c, the mean rate is $\mu = c/100$, with uncertainty $\sqrt{100c}/100 = \sqrt{c/100}$. We can see that the estimates from the two interval-lengths are consistent; the greatest difference between means is only 1.5σ (from the $\sim$1Hz data).

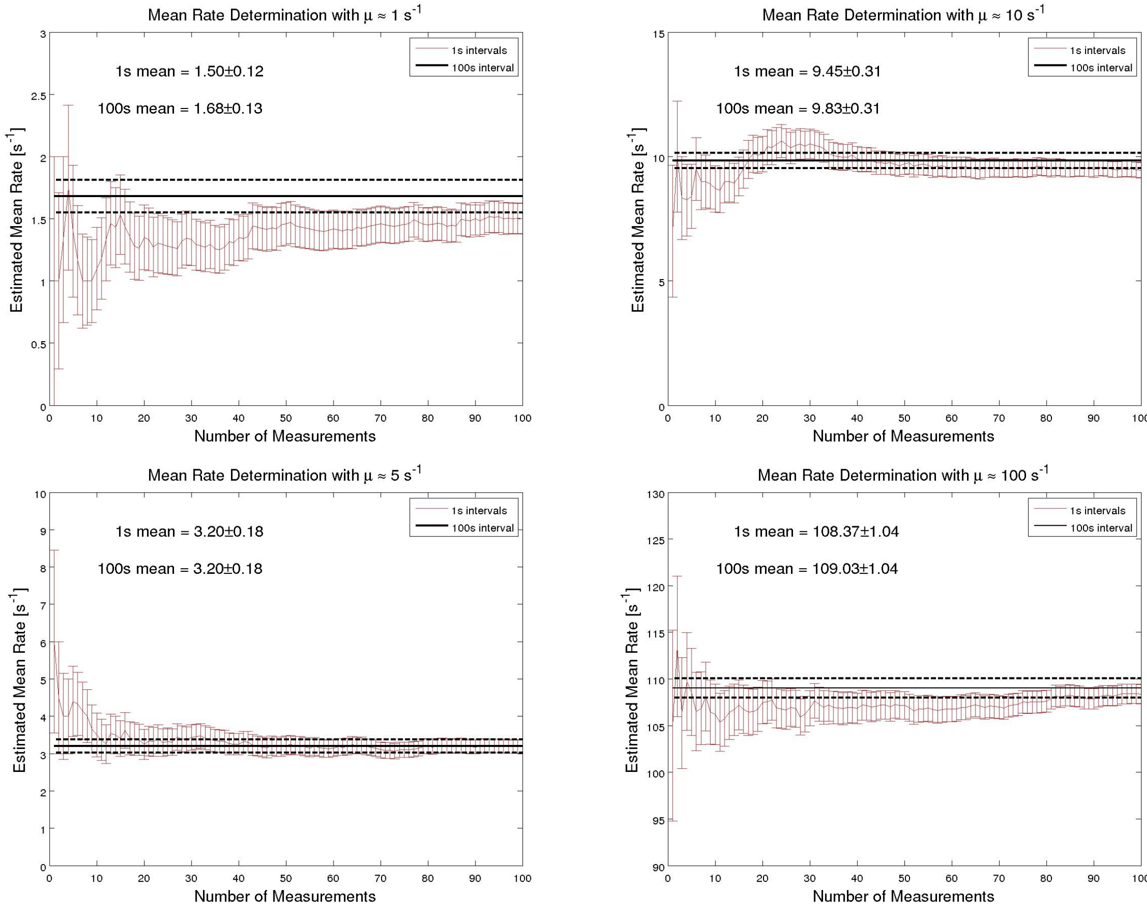

FIG. 5: For each desired rate, the running mean (cumulative average) from the 1s trials is shown in red, with the estimate from the 100s trial overlaid in black. Errorbars calculated as described above.

We also fit the 1s data directly to Poisson curves as shown in Figure 6. The higher rates are solid fits with p-values of .86, .84, and .93, while the ∼1Hz data does not wear the Poissonian well, generating a p-value of .13. We will discuss what could have affected the low-frequency data in our sources of error analysis.

IV. ERROR ANALYSIS

Conveniently this experiment is quite robust against error. If our model is correct, then we would expect any source of radiation to obey Poisson statistics. And since Poisson distributions always combine to produce more Poisson distributions, any background radiation contributing to our counts will change the mean rate we measure, but not break the Poisson character of our measurements. In fact, most sources of systematic error, eg. uncertainties in apparatus parameters, only change the mean rates we measure. But actual mean rates are not the goal of the experiment, and, since we tuned the experiment to acheive certain desired approximate rates anyway, these values have no significance.

There is one source of systematic error which could in principle disturb the distribution. The photomultiplier tube will have some deadtime after any pulse, during which it cannot activate another pulse. This effect is obviously a more important concern at higher count rates, so let us estimate its impact on the 100Hz data. We can see from the oscilliscope that the timescale for the rise and fall of a peak is about $\sim 2\mu$s. Over the entire 100 seconds of a run, we expect 100Hz $\times$ 100s $=$ 10000 dead intervals, which yields a total dead time of $\sim$ 20ms. Assuming the counts are actually Poisson with mean rate $\sim$ 100Hz, the expected number of counts in that interval is 100Hz $\times$ 20ms $=$ 2, with an uncertainty ± 1.4. So our estimate of the mean rate (divide by 100 seconds) would have a systematic bias of about $\sim$ .02Hz with systematic uncertainty around $\sim$ .014Hz. On the scale of 100Hz, this is already much smaller than our statistical uncertainty, and the effect becames less and less significant at lower rates. So deadtime is not a serious problem for our experiment.

As we have discussed, background radiation is not a worry–*unless* the radiation changes during the experiment. If the mean rate of background radiation changed, that would contribute non-Poisson noise. This is plausible if radiation came from a mobile source near the experiment, such as, for instance, the experimenters themselves. We can estimate the rate of counts from the bodies of the experimenters to determine whether we could have affected the results. A typical human body contains [1] about $.1\mu$Ci of ^{40}K, which corresponds to 3.7×10^3 decays per second. 11% of those, or 400 decays per second, emit gamma radiation (at an energy level beyond our highest discriminator level). Pretending, for ease of calculation, the the human body is a point source of radiation about .5m from the detector, we can calculate the solid angle for radiating into a a scintillator crystal [2] of area $(.051m)^2$, which leads to an expectation of about .33 counts per second into the detector. This is on the scale of our smallest measurements (the troublesome 1Hz data), so, by moving around during the run, we could actually have contributed significant non-Poisson noise, enough to significantly disturb the 1Hz distribution. Data at higher rates are less afffected.

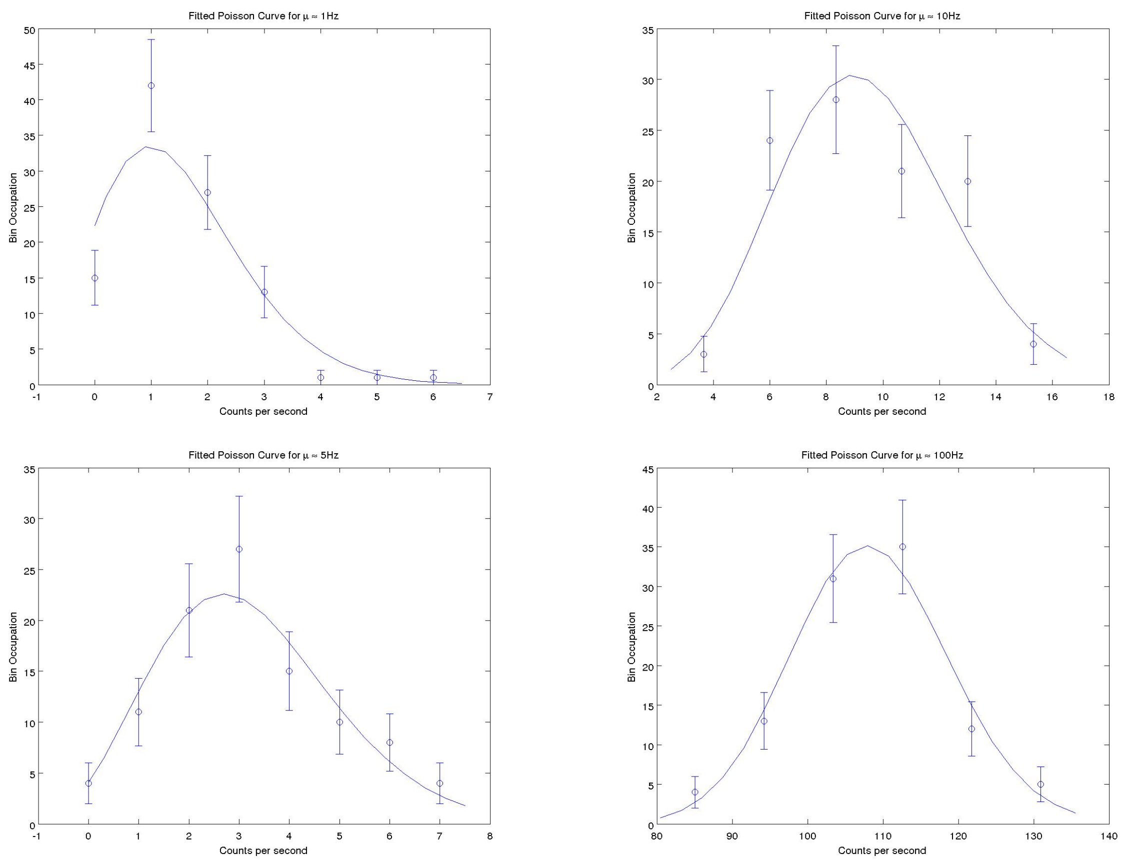

FIG. 6: Poisson fits for each desired rate.

V. CONCLUSION

In conclusion, we have discussed properties of an incredibly common and useful distribution, we have found that gamma radiation seems to obey this distribution, and we have come across an interesting source of error in a standard introductory experiment.

[1] Frame, Paul. *General Information about K-40.* 20 January 2009. http://www.orau.org/ptp/collection/consumer%20products/potassiumgeneralinfo.htm.

[2] Detector datasheet: http://www.canberra.com/products/detectors/pdf/Model-802-SS-CSP0232.pdf