Non-demolition Photon Detection

Samuel James Bader
MIT Department of Physics, (4-304) 77 Massachusetts Ave., Cambridge, MA 02139
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This paper reviews several experiments of microwave cavity quantum electrodynamics in which quantum non-demolition (QND) measurements of photons played a role. It covers QND first in single photon fields, then in larger fields, discusses how to determine coherence information, and finally covers the basic structure of a state preparation feedback loop.

I. INTRODUCTION

Advances in cavity quantum electrodynamics and its potential for use in optical quantum computing have greatly increased both the desirability and feasibility of precise control and measurement of cavity photon fields. Various techniques already exist for detecting fields to the precision of a single photon, such as the avalanche photodiode, and photomultiplier tube, and there has even been research on using quantum-dot-based field effect transistors for this purpose [1]. Unfortunately, all these methods destroy the photons in measurement. However, such limitations are not fundamental to quantum mechanics. The techniques of cavity QED combined with atom interferometry provide an ingenious method of resolving the photon number of a cavity field without destroying its projected state.

Quantum non-demolition (QND) measurements, as such experiments are referred to more broadly, allow one to track the evolution of an individual realization of a quantum system. This forms the basis of many potentially useful techniques for both the future of quantum computing and fundamental tests of quantum physics. In this paper, we shall discuss the progress in QND experiments, beginning with the QND detection of a single photon, and generalize to the determination of larger photon numbers, discuss phase information, and finally cover a feedback loop implemented in such systems for state preparation.

II. QND WITH A SINGLE PHOTON

A. Theory

Before detailing single-photon measurements, it is prudent to mention a common method for optical QND measurements of *macroscopic* light intensities, as this will provide an insightful analogy for the discussion. This method involves two beams: a 'signal' beam whose intensity we would like to measure, and an auxiliary 'meter' beam. The meter beam is split into two components. One component is sent through a non-linear Kerr medium (a material whose refractive index depends on light intensity) *simultaneously with the signal beam.* The phase shift of the meter beam passing through the medium then depends on the intensity of the signal beam. Thus, upon recombining this component of the meter beam with the other, the resulting interference pattern will be shifted, and one can determine the signal intensity [2, 3].

By analogy, for single-photon measurements, we will simply replace the meter beam and optical interferometry with single atoms and atomic interferometry [2]. The signal field represents one mode of an optical cavity, through which we send one 'meter' atom at a time.

The three relevant states of the atom we shall refer to as $|i\rangle$, $|g\rangle$, and $|e\rangle$ (in order of increasing energy). We will be injecting atoms in superpositions of $|i\rangle$ and $|g\rangle$ into a cavity resonant with the $|g\rangle \Leftrightarrow |e\rangle$ transition.

So an atom in the state $|i\rangle$ passes largely unaffected through the cavity (far off-resonant from every transition), and an atom in the state $|g\rangle$ experiences a Jaynes-Cummings Hamiltonian (JCH) connecting it to $|e\rangle$:

$$\begin{aligned}\hat{H}_{JC} &= \hat{H}_{atom} + \hat{H}_{field} + \hat{H}_{int} \\ &= \frac{\hbar\omega_{ig}}{2}\hat{\sigma}_z + \hbar\omega\hat{a}^\dagger\hat{a} + \frac{\hbar\Omega}{2}(\hat{a}\hat{\sigma}_+ + \hat{a}^\dagger\hat{\sigma}_-)\end{aligned}$$

where Ω is the vacuum Rabi frequency, ω is the atomic transition frequency resonant with the cavity, the a operators are photon creation and annihilation, and the Pauli σ operators act on the atomic states.

If the atom enters in $|g\rangle$ and the photon field begins in the state $|0\rangle$, then the interaction term vanishes (because JCH uses the rotating-approximation, an empty cavity will not connect $|g\rangle$ to $|e\rangle$), and the atom is unaffected. However, if the atom enters in the state $|g\rangle$ and the photon field begins in the state $|1\rangle$, the atom will undergo vacuum Rabi oscillations $|g,1\rangle \Leftrightarrow |e,0\rangle$. The state of the atom-field system will thus be given by

$$\cos(\Omega t/2)\,|g,1\rangle + \sin(\Omega t/2)\,|e,0\rangle$$

As can be read from the above expression, if the interaction time with the cavity is $2\pi/\Omega$, then the atom completes a full Rabi oscillation back to $|g\rangle$, having absorbed and reemitted the photon, but in the process the state has picked up a phase factor of -1. Thus we have coupled the photon number of the cavity to the phase of the atomic state, without changing the photon number.

Before continuing, let us recapitulate the effect which we just derived that the interaction has on the following initial states:

$$\begin{aligned} |i,0\rangle &\rightarrow & |i,0\rangle \\ |i,1\rangle &\rightarrow & |i,1\rangle \\ |g,0\rangle &\rightarrow & |g,0\rangle \\ |g,1\rangle &\rightarrow & -|g,1\rangle \end{aligned}$$

We can measure this phase shift via the following Ramsey interferometry scheme [2]. First, a $\pi/2$ pulse at frequency ν, near resonant with the $|i\rangle \Leftrightarrow |g\rangle$ transition, prepares, from atoms in $|i\rangle$, the superposition

$$|i\rangle \rightarrow |+\rangle = \frac{1}{\sqrt{2}}(|i\rangle + |g\rangle)$$

Then the atom passes through the cavity, and undergoes the aforementioned Rabi evolution: it will flip the phase of the excited state if the photon number is one, and have no effect if the photon number is zero. (Let us refer to the phase-flipped superposition as $|-\rangle$). Taking the initial cavity state to be $\alpha_0 |0\rangle + \alpha_1 |1\rangle$, we have

$$|+\rangle \otimes \{\alpha_0 |0\rangle + \alpha_1 |1\rangle\} \rightarrow \alpha_0 |+,0\rangle + \alpha_1 |-,1\rangle$$

Subsequently, another $\pi/2$ pulse (phase coherent with the first) converts these superpositions back to basis states:

$$\alpha_0 |+,0\rangle + \alpha_1 |-,1\rangle \rightarrow \alpha_0 |i,0\rangle + \alpha_1 |g,1\rangle$$

Then the atom passes into a state detector, which should detect $|i\rangle$ if the photon number is 0, $|g\rangle$ if the photon number is 1.

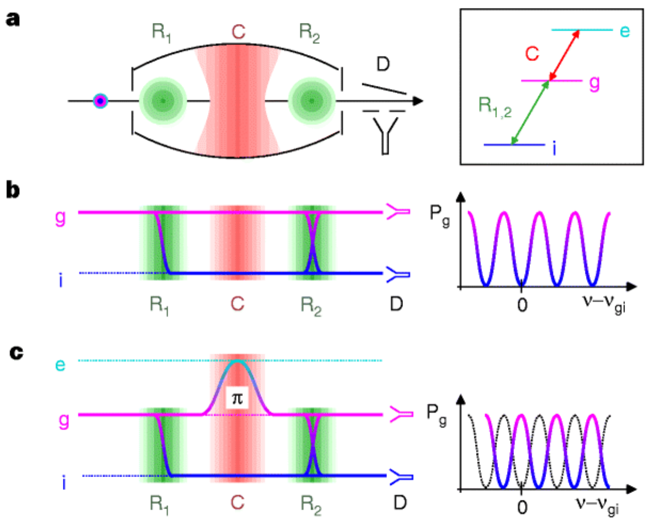

FIG. 1: The Ramsey interferometry for single-photon detection. R_1 and R_2 are the two Ramsey pulses and C is the cavity. (a) Schematic and level structure. (b) The atomic state with no photon in the cavity. (c) The same, with a single photon. Also shown are the Ramsey fringes with variation in ν. Figure from [2].

B. Experimental Considerations

Two further experimental details of this procedure are worth noting. First, the meter atom is typically prepared in a circular Rydberg state, that is, one of high principal quantum number n and maximum orbital and magnetic quantum numbers (ie $l = |m| = n-1$). In the specific experiment of [2] discussed here, these numbers are $n =$ 49, 50, and 51 for i, g, and e respectively. The reason for the choice of circular Rydberg states [4] is that they have **large dipole matrix elements** (dipole elements scale as n^2), and **long lifetimes** (since frequency differences go as n^{-3} and dipole element goes as n^2, the decay rate goes as $(n^{-3})^3(n^2)^2 = n^{-5}$). For [2], the lifetime of e is 30ms.

Secondly, the Stark effect can be used to adjust the interaction time of the atom with the cavity, instead of relying on velocity selection alone. Applying a separate DC electric field across the mirrors of the cavity, one can fine-tune the atomic transition frequency, choosing the length of time over which it should be resonant with the cavity.

With these experimental methods and the previous theory discussion, we have completed the QND measurement of a single photon.

III. MULTIPLE PHOTON QND

A. Off-resonant interaction

The method described above only works when the photon field is restricted to $n = 0, 1$. The $\sqrt{n}$ dependence of the photon annihilation operator in the Jaynes-Cummings Hamiltonian implies that the rate of flopping between $|g\rangle$ and $|e\rangle$ in the presence of a photon field which begins in state n is not in general Ω but rather $\sqrt{n}\Omega$. And thus, the proper interaction time, to guarantee that the atom exactly absorbs and re-emits the photon depends on the photon number *which we hope to measure*, which makes a non-destructive measurement impossible.

However, there is an different source of phase shift we can probe. From now on, we shall only require two atomic states; let us choose $|g\rangle$ and $|e\rangle$. Should we significantly detune the cavity by $\delta = \omega - \omega_{eg}$ from the transition frequency ω_{eg}, then amplitude of Rabi oscillations will be negligible, such that we can neglect the small probability of photon absorption. However, the interaction of the cavity will still Stark shift the energies of the ground and excited states such that, over the interaction time, a phase shift will accumulate between the two [2, 5]. This approach was actually described before the realization of the single-photon measurement described above, but was more technically difficult to implement [2].

We can derive this energy shift from second-order perturbation theory. If an atom is sent into the n-photon cavity in state $|g\rangle$ such that the total state is $|g,n\rangle$, then it will be strongly coupled to the state $|e,n-1\rangle$ by the

JCH interaction, since these two states are separated only by $\hbar\delta$ in energy. The matrix element connecting the two is $\hbar\Omega\sqrt{n}/2$, so the second-order energy shift, *taking into account only this coupling*, is

$$\frac{|H_{g,n;e,n-1}|^2}{E_{e,n-1} - E_{g,n}} = \frac{\hbar\Omega^2 n}{4\delta}$$

Similarly, if an atom enters the n-photon cavity in the state $|e\rangle$, then the total state, $|e, n\rangle$, is most closely coupled to $|g, n+1\rangle$. The energy separation is $-\hbar\delta$, and the matrix element is $\hbar\Omega\sqrt{n+1}/2$, so the second-order shift is

$$\frac{|H_{e,n;g,n+1}|^2}{E_{g,n+1} - E_{e,n}} = -\frac{\hbar\Omega^2(n+1)}{4\delta}$$

Comparing the above results, the perturbed energy difference between $|e, n\rangle$ and $|g, n\rangle$ is then

$$|\Delta E| = \frac{\hbar\Omega^2}{2\delta}\left(n + \frac{1}{2}\right)$$

So if the atom interacts with the cavity for time t, there will be an accumulated phase difference of

$$\frac{\Omega^2 t}{2\delta}\left(n + \frac{1}{2}\right)$$

This result we have derived here agrees with the expression stated in [5]. Felicitously, the phase is linear in photon number. And, the phase shift per photon $\Phi = \frac{\Omega^2 t}{2\delta}$ is adjustable via the detuning (though greater care must be taken with higher order corrections for smaller detunings compared to the Rabi frequency). We can use the same Ramsey technique as before to measure this phase shift.

Note: the trivial part of the phase shift (independent of n) will be ignored for the remainder of the paper. We can just assume that the second Ramsey pulse is offset by $\Omega^2 t/4\delta$ so that this term is shifted away. (This is in addition to any other phase offsets we add to the second Ramsey pulse in the upcoming section.)

B. Procedure

Suppose we wish to use this method to distinguish between $n = 0$ and $n = 1$ as before. In principle, only one atom, again, is necessary. The first Ramsey pulse creates the $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$ superposition. We choose δ such that the phase shift per photon is π. Then the second pulse converts the unaffected superposition ($n = 0$) to $|e\rangle$ and the phase-shifted superposition ($n = 1$) back to $|g\rangle$. We recover our single-photon results.

To be more general, let us assume we wish our measurement to distinguish all the possible Fock states (states of definite photon number) from 0 to N, where N is some reasonable upper bound on the photon number of the cavity. For instance, if we inject into the cavity a small classical coherent field with mean value of $\bar{n} = 3$, then n should follow a Poisson distribution; if we set $N = 7$ for the measurement procedure, our bound is correct 98.8% of the time.

To start, the procedure [5] assumes no knowledge of the cavity state. The initial guess is thus a uniform probability distribution for the photon number

$$P^{(0)}(n) = 1/(N+1), \quad 0 \le n \le N$$

First the phase shift per photon of the cavity is adjusted to $\Phi = 2\pi/(N+1)$ so that each Fock state will result in a maximally distinct phase. As, before an atom is taken from $|g\rangle$ to $\frac{1}{\sqrt{2}}(|g\rangle + |e\rangle)$ by the initial Ramsey pulse. The atom then passes through the cavity and entangles with each cavity Fock state $|n\rangle$ such that the phase shift of the excited atomic state is Φn.

Now we would like to be able to ask, not just whether the relative phase has picked up a minus sign as in the single photon case, but we would like to be able to check whether it is any arbitrary ϕ. In general, if the second Ramsey pulse is set to ϕ out of phase with the first pulse, it will convert

$$|+\rangle_\phi = \frac{1}{\sqrt{2}}(|g\rangle + e^{i\phi}|e\rangle) \to |e\rangle$$
$$|-\rangle_\phi = \frac{1}{\sqrt{2}}(|g\rangle - e^{i\phi}|e\rangle) \to |g\rangle$$

So we can use this measurement to probe all the phase shifts corresponding to different photon numbers. However, we cannot completely determine the phase of the atomic state by one measurement. If we measure for any offset ϕ, in general there are multiple n such that $\frac{1}{\sqrt{2}}(|g\rangle + e^{i\Phi n}|e\rangle)$ has a component along the $|+\rangle_\phi$ state. So detecting $|e\rangle$ after the second Ramsey pulse does not narrow the possibles phase to just some ϕ. However, it does make certain phases more likely than others (and it eliminates the possibility that the phase is $\phi + \pi$).

So we will have to send in multiple atoms (one at a time still), and measure a phase for each to narrow down the actual phase shift being produced by the cavity. Each measurement provides more information to condition the probability of a given Fock state. Let us now examine the implications of each measurement upon the state of the cavity.

C. Pinning down the Fock state

For the mth measurement, let the current state of the cavity be given by

$$\sum_{n=0}^{N} \alpha_n^{(m-1)} |n\rangle$$

Upon sending in the mth atom, we have the entangled state

$$\sum_{n=0}^{N} \frac{\alpha_n^{(m-1)}}{\sqrt{2}} \{|g\rangle + e^{i\Phi n}|e\rangle\} \otimes |n\rangle$$

Let us offset the second Ramsey pulse by ϕ, in which case it is useful to rewrite the above expression in terms of $|+\rangle_\phi$ and $|-\rangle_\phi$.

$$\sum_{n=0}^{N} \alpha_n^{(m-1)} \{\frac{1}{2}(1+e^{i(\Phi n-\phi)})\,|+\rangle_\phi + \frac{1}{2}(1-e^{i(\Phi n-\phi)})\,|-\rangle_\phi\} \otimes |n\rangle$$

Trivially, the second Ramsey pulse converts this to

$$\sum_{n=0}^{N} \alpha_n^{(m-1)} \{\frac{1}{2}(1+e^{i(\Phi n-\phi)})\,|e\rangle + \frac{1}{2}(1-e^{i(\Phi n-\phi)})\,|g\rangle\} \otimes |n\rangle$$

Now the detection will collapse the entangled state into either the $|e\rangle$ or $|g\rangle$ component. So, defining $j = 0, 1$ for detecting $|e\rangle$, $|g\rangle$ respectively, we read off that the new coefficient of the nth Fock state is

$$\alpha_n^{(m)} = \frac{1}{\sqrt{Z}} \alpha_n^{(m-1)} \frac{1}{2}(1 + e^{i(\Phi n - \phi + j\pi)})$$

where Z takes care of the overall normalization (and equals, up to a phase factor, the conditional probability for detection of whichever state had just been detected). We do not care about the phases of the Fock states, only their probabilities, so let us examine how this measurement has affected the probability that the cavity is in the nth state.

$$\begin{aligned} P^{(m)}(n) &= |\alpha_n^{(m)}|^2 \\ &= \frac{1}{Z}|\alpha_n^{(m-1)}|^2 \frac{(1+\cos(\Phi n - \phi + j\pi))}{2} \\ &= \frac{1}{Z} P^{(m-1)}(n) \frac{(1+\cos(\Phi n - \phi + j\pi))}{2} \end{aligned}$$

Let us just run three checks that this makes sense.

1. If we have the offset ϕ and we detect the excited state ($j = 0$), then the probabilities are now more concentrated in the states n such that $\Phi n \approx \phi$.

2. We recapture the trivial single photon case: with $\Phi = \pi$ and $\phi = 0$, we have all the probability in $n = 0$ if $|e\rangle$ is detected, and all the probability in $n = 1$ if $|g\rangle$ is detected.

3. If we detect the excited state, then there is zero probability that the phase shift equals $\phi + \pi$, as claimed earlier.

So, after M measurements, we find by induction that

$$P^{(m)}(n) = P^{(0)}(n) \prod_{m=1}^{M} \frac{(1+\cos(\Phi n - \phi_m + j_m\pi))}{2Z_m}$$

This agrees with the results derived by [6] by a different approach: the language of Bayesian conditional probabilities. It is worth noting that classical probability theory for the cavity state, as used in [6], justifiably achieves the same result as our treatment of the quantum superposition. After the atomic measurement, the state of the cavity is not collapsed, so the contributions to detection probabilities from different Fock states do not interfere (because the "which-way" evidence is still remaining in the cavity).

Although the initial state and probabilities $P^{(0)}(n)$ of the cavity are unknown, the measurements will tend to concentrate the cavity in a finite number of steps into some single Fock state in agreement with the measured values of the phase shift. As mentioned, the procedure for measurement simply approximates a flat distribution for the initial probabilities, and, after many atoms have passed, the measurements should indicate the correct collapsed Fock state. And we have thus completed a non-demolition measurement of the arbitrary photon number of a cavity.

D. Repeated Measurements

In [6], each "single measurement" is implemented via a sequence of 110 atoms, in a 51.1 GHz Fabry-Pérot cavity with a long damping time of $T_c = .130$s, and the cavity is cooled to .8K such that the expected thermal photon number is $n_t = .05$. Each sequence takes on the order of $T_m = .026$s, so the measurement can be repeated multiple times on the same realization of the field, *with the same result expected.* Since the goal is that the collapsed state of the cavity survives each measurement, this repeatability of measurements on a single realization of a system is the fundamental property of QND.

The plots in Figure 2 each measure an individual realization of an initial coherent "signal" field for $\sim 29{,}000$ atoms. Each sequence of 110 atoms is taken to be a full measurement, after which the algorithm re-initiates its estimate of the probabilities to a flat distribution and begins calculating again (so each plot contains ~ 26 independent measurements). The expected photon number as estimated by the algorithm is then plotted against time.

The plots quickly converge to stable $\langle n\rangle$ values at definite photon numbers and plateau for a time which typically corresponds to ~ 2 measurements. Damping results in slow relaxation of the field, and because the measurements frequently collapse the cavity into Fock states, this damping appears as a series of quantum jumps, stepping the photon number down one at a time to the vacuum. (As can be seen in the magnified inset of the first plot, the measurement procedure takes some time, about $\sim .01$s, to adjust to each quantum jump.) Additionally, thermal fluctuations provide occasional kicks to the photon number, most notably in the lower rightmost plot where $n = 1$ jumps up entirely to $n = 2$ [6].

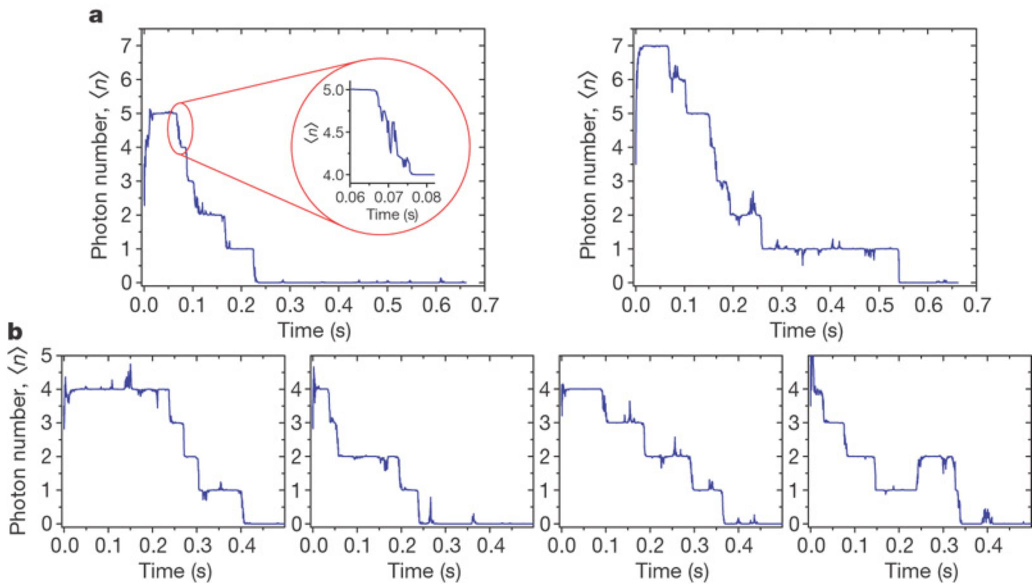

FIG. 2: Expected photon number vs time for six independent realizations of the initial cavity field [6].

E. Field state reconstruction

By repeating this measurement on many separate realizations of an initial field produced by the same process, we could reconstruct the photon number statistics of the field. For instance, [6] generated the following histogram (Figure 3) from 2000 measurement sequences (of the type described) above on a small coherent field.

In Figure 3 we see integer peaks corresponding to measurement frequencies of the different Fock states with a small background noise of measurement sequences which either did not collapse in time or were interrupted by decay. As expected, $\langle n \rangle$ follows a Poissonian distribution. This plot can be fitted to the the evolution of an initial coherent state of $\bar{n} = 3.82$ which has decayed for half the measurement time. Even taking into account the decay, there is a slight excess of probability in the vacuum state, but this is well understood: the measurement only picks out a Fock state by its phase shift, so it only distinguishes photon numbers modulo $N + 1$ (which is in this case 8). The state of highest probability that is above the arbitrary $N = 7$ bound in this plot is the $n = 8$ state, which will be detected then, as the $n = 0$ state, slightly skewing the distribution.

However this only measures the probabilities of the states, not the coherences between them [5]. In the language of density operators, this measurement captures only the diagonal elements ρ_{nn} of the field's density matrix. The phase coherence information (ie the off-diagonal elements of the density matrix) can be measured by mixing the signal field with reference coherent states of known complex amplitude α.

The effect of pulse-injecting a classical coherent state into the cavity is to supply a phase-translation to the cavity field, given by the Glauber translation operator: $D(\alpha) = e^{\alpha \hat{a}^\dagger - \alpha^* \hat{a}}$. The density operator afterword is then

$$\rho^{(\alpha)} = D(\alpha) \rho D(-\alpha)$$

$D(\alpha)$ will mix the off-diagonal elements into the diagonal of the Fock basis. Via this interference between the signal field and the injected field of known phase, signal phase information is converted into signal photon number information. So sampling at many values of α, and reconstructing the photon number distribution for each as before yields constraints for the off-diagonal elements [5], which can be solved by a quantum tomography technique known as the method of maximum entropy [7], outside of our current scope.

And thus, with the techniques discussed up to this point, we have achieved our goal: we can measure the density operator of an arbitrary initial photon field. But we can push these methods even further.

IV. BEYOND MEASUREMENT

The result of measurement is a cavity field collapsed into a random photon number. If we could choose that number, then these techniques would provide a method for generating arbitrary Fock states on demand, a useful tool for quantum optics. Although the results of the measurement as given are random, researchers in [8] presented and realized a procedure for deterministically steering the cavity into a predetermined state.

The idea is a feedback loop: we can begin by injecting a classical field whose $\bar{n}$ is the desired value. On each iteration of the loop, we complete QND measurement, update the density matrix with the new information, and then inject a classical field with an amplitude chosen so as to bump the cavity state toward the desired Fock state.

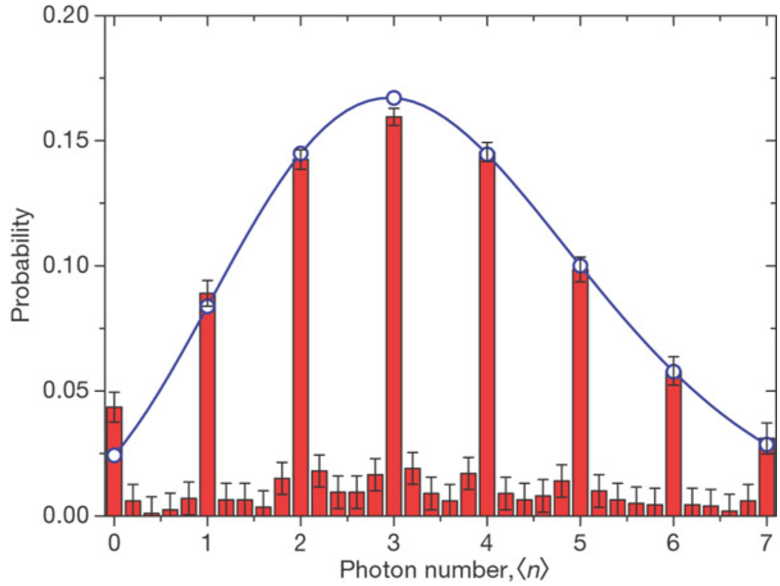

FIG. 3: Photon number statistics of a classical coherent field [6]

The feedback loop could then continue to monitor the state and protect it from decay by supplying more controlled injections whenever it detects a quantum jump. In this way, arbitrary Fock states can be generated on demand and preserved for timescales far beyond their cavity lifetime.

V. CONCLUSION

The fundamental principle which allows for such feedback systems to operate is the non-demolition nature of the quantum measurements [5]. We have come a long way in this review of QND photon techniques, from the detection of a single photon, to mapping out arbitrary cavity states and even creating and protecting the fragile Fock states. With these skills under our belt, a whole new world of cavity quantum electrodynamic manipulations and measurements have become accessible to experiment, from fundamental probes of the quantum regime, to new testing grounds for quantum information science.

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